Tree/Stack Splitting for Medium Access Control in Wireless Networks with Concurrent Multiple Packet Reception

Rung-Hung Gau
Associate Professor
Department of Computer Science and Engineering
National Sun Yat-Sen University
Outline

- Medium Access Control
- Tree/Stack Splitting Algorithm
- Multiple Packet Reception
- Analytical Results
  - Network Throughput
  - Average System Delay
- Simulation Results
- Conclusions
Broadcast Downlink vs. MAC Uplink

\[
\begin{align*}
H_1 & \rightarrow + \rightarrow y_1 \\
H_2 & \rightarrow + \rightarrow y_2 \\
\vdots \\
H_K & \rightarrow + \rightarrow y_K \\
\end{align*}
\]

\[
\begin{align*}
H_1^\dagger & \rightarrow u_1 \\
H_2^\dagger & \rightarrow u_2 \\
\vdots \\
H_K^\dagger & \rightarrow u_K \\
\end{align*}
\]

\[
\begin{align*}
+ & \rightarrow v \\
\end{align*}
\]

\[
\begin{align*}
t_1 \\
t_2 \\
\vdots \\
t_K \\
\end{align*}
\]
Medium Access Control

- Distributed Random Access
  - Aloha
  - Tree/Stack Splitting
  - CSMA
  - IEEE 802.11 DCF
  - IEEE 802.15.4 Contention Access Period

- Centralized
  - IEEE 802.11 PCF
  - IEEE 802.15.4 Guaranteed Time Slots
  - IEEE 802.16
System Models

- One access point (AP) and N nodes
- Maximum queue size at each node is B
- Time is divided into slots
- The packet transmission time is a slot
- We focus on uplink shared medium
Conventional Collision Channel

- $(0,1,e)$ channel model is widely used
  - If two or more nodes simultaneously transmit packets, a collision occurs and the AP does not receive any packets
- For wireless networks, a minor revision is used
  - Packet error probability is included
Related Works

Related Works


Related Works


• Many more to come…
The Tree/Stack Splitting Algorithm

cost : 7 slots
Stack Evolution at Node A
Stack Evolution at Node B
Stack Evolution at Node C
On System Evolution

$X_{n-1} = 4$

$X_n = 2$

$X_{n+1} = 3$

$X_{n+2} = 1$

$Y_{n-1}$

$Y_n$

$Y_{n+1}$

packet arrival

time
Notations

- $X_k$: the number of nodes that have packets to transmit at the beginning of $k$-th CRC (Collision Resolution Cycle)
- $X_k$ is called the order of the $k$-th CRC
- $Y_k$: the length of $k$-th CRC
Expected length of a CRC

- $T(n) = E[Y_k | X_k = n]$: the average length of a $n$-th order CRC in terms of time slots
- Recall that $M$ is the channel capacity
- If less than or equal to $M$ nodes concurrently transmit, the AP receives all of them.
- If more than $M$ nodes concurrently transmit, the AP receive none of them.
\[
T(n) = 1 + \sum_{k=0}^{n} \binom{n}{k} \left(\frac{1}{2}\right)^n (T(k) + T(n-k)), \quad n > M
\]

\[
T(0) = T(1) = \cdots = T(M) = 1, \quad n \leq M
\]
Markov Chain Modeling

• $X_1, X_2, X_3, \ldots$ form a discrete-time Markov chain
Is it possible to simultaneously receive two or more packets from distinct nodes using a single frequency band?
Multiple Packet Reception Channel

- Uplink channel is a Multiple-Packet-Reception channel represented by a matrix $R$
- $R_{i,j}$ is the probability that the AP successfully receives $j$ packets when $i$ nodes transmit in a time slot, $1 \leq i, j \leq N$
- We focus on the case in which the channel matrix $R$ is a $N$ by $N$ diagonal matrix with rank $M$
$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
Multiple Packet Reception
Communications Technology and Theory

- Network Information Theory
- CDMA with Multi-User Detection
- Multiuser MIMO (Multiple Input Multiple Output) + Space-Time Coding
- SIC (Successive Interference Cancellation)
Information Theoretical MAC Capacity Region (two users, each has one transmitting antenna)

\[ R_1 + R_2 \leq \log |I + H_1^\dagger P_1 H_1 + H_2^\dagger P_2 H_2| \]
Information Theoretical MAC Capacity Region (two users, each has multiple transmitting antenna)
CDMA Multi-User Detection (1)
CDMA Multi-User Detection (2)
OFDM + Space-Time Coding
Expected length of one CRC

average # of time slots required for transmitting packets

- M = 2
- M = 3

number of nodes

<table>
<thead>
<tr>
<th>time slots</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

0  | 3  | 6  | 9  | 12 |

2 | 4  | 8  | 12 | 16 |

20 | 24 | 28 | 32 | 36 | 40 |

28
State Transition Probability

\[ P\{X_{n+1} = m_{n+1} | X_n = m_n \} \]

\[ = \sum_{k=1}^{\infty} P\{X_{n+1} = m_{n+1} | Y_n = k \} \times P\{Y_n = k | X_n = m_n \} \]
State Transition – Type 1

- $q$: the probability of packet arrival at one node in a time slot.
  - $1-q$: no packet arrival at one node in a time slot.
  - $(1-q)^k$: no packet arrival at one node in $k$ time slots.
  - $1 - (1-q)^k$: at least 1 packet arrival at one node in $k$ time slots.
- All nodes have the same value of $q$.

\[ P\{X_{n+1} = m | Y_n = k\} = \binom{N}{m} \times (1 - (1-q)^k)^m \times ((1-q)^k)^{N-m} \]
State Transition – Type 2

- let $\Phi(m,k) = P\{Y_n=k \mid X_n=m\}$
- recursive splitting
\[ \Phi(m, k) = P\{Y_n = k | X_n = m\} \]
\[ = \sum_{nL=0}^{m} \binom{m}{nL} \left(\frac{1}{2}\right)^m \sum_{tL=1}^{k-2} \Phi(nL, tL) \times \Phi(m - nL, k - tL - 1) \]
Throughput analysis

- throughput = average successfully received packets per time slot

- $v(k)$: steady-state probability of $X_n=k$

- $l(k)$: average length of a $k$-th order CRC

$$\lambda_D = \frac{\sum_{k=0}^{N} \frac{v(k)}{v(0)} \cdot k}{\sum_{k=0}^{N} \frac{v(k)}{v(0)} \cdot l(k)}$$
Delay analysis

- $s_1(n)$: the average number of busy servers in a collision resolution cycle of order $n$
- $s_2(n)$: the average queue size of a node in a collision resolution cycle of order $n$
The Average Number of Busy Servers in a CRC

\[ l(n) \cdot s_1(n) = \sum_{m=0}^{n} C^n_m \cdot \left( \frac{1}{2} \right)^n \]

\[ \cdot \left\{ 1 \cdot n + \left[ l(m) \cdot s_1(m) + l(m) \cdot (n - m) \right] 
+ l(n - m) \cdot s_1(n - m) \right\}. \]
The Average Number of Busy Servers in a Regenerative Cycle

\[
E[W] = \frac{\sum_{k=0}^{N_t} \frac{v(k)}{v(0)} \cdot s_1(k) \cdot l(k)}{E[L]}
\]

\[
= \frac{\sum_{k=0}^{N_t} v(k) s_1(k) l(k)}{\sum_{k=0}^{N_t} v(k) l(k)}.
\]
The Average Queue Size at a Node in a CRC

\[ l(n) \cdot s_2(n) \]

\[ = \sum_{m=2}^{\infty} \Phi(n, m) \sum_{k=1}^{m-1} \left[ (1 - q)^{k-1} \cdot q \right] \cdot (m - k). \]
The Average Queue Size at a Node in a Regenerative Cycle

\[
E[Q] = N_t \cdot \frac{\sum_{k=0}^{N_t} \frac{v(k)}{v(0)} \cdot s_2(k) \cdot l(k)}{E[L]}
\]

\[
= N_t \cdot \frac{\sum_{k=0}^{N_t} v(k) s_2(k) l(k)}{\sum_{k=0}^{N_t} v(k) l(k)}
\]
The Average System Delay

\[ E[D] = \frac{E[W] + E[Q]}{\lambda_D}. \]
Network Throughput, 10 nodes, B=1

![Graph showing network throughput with aggregated packet arrival rate on the x-axis and throughput on the y-axis.](image)
Network Throughput, 50 nodes, $B=1$
Network Throughput, 100 nodes, B=1
Average System Delay, 100 nodes, B=1
<table>
<thead>
<tr>
<th>total arrival rate</th>
<th>throughput</th>
<th>lower bound of the 95 % confidence interval</th>
<th>upper bound of the 95 % confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.998177</td>
<td>1.995379</td>
<td>2.000975</td>
</tr>
<tr>
<td>4</td>
<td>3.996476</td>
<td>3.992554</td>
<td>4.000398</td>
</tr>
<tr>
<td>8</td>
<td>7.995147</td>
<td>7.989720</td>
<td>8.000573</td>
</tr>
<tr>
<td>10</td>
<td>9.835345</td>
<td>9.828388</td>
<td>9.842302</td>
</tr>
<tr>
<td>12</td>
<td>8.015199</td>
<td>8.009847</td>
<td>8.020550</td>
</tr>
<tr>
<td>14</td>
<td>7.682196</td>
<td>7.677497</td>
<td>7.686895</td>
</tr>
<tr>
<td>16</td>
<td>7.310317</td>
<td>7.306137</td>
<td>7.314497</td>
</tr>
<tr>
<td>20</td>
<td>7.152182</td>
<td>7.148292</td>
<td>7.156073</td>
</tr>
</tbody>
</table>
Network Throughput, 10 nodes, B=2

![Graph showing network throughput for B=2 with 10 nodes. The graph plots throughput against aggregated packet arrival rate, with different markers representing different scenarios and simulation results.]
TABLE 2
The 95 Percent Confidence Intervals Based on the Method of Batch Means for Throughput when \( N_t = 10, M = 2, \) and \( B = 2 \)

<table>
<thead>
<tr>
<th>total arrival rate</th>
<th>throughput (analytical)</th>
<th>throughput (simulation)</th>
<th>lower bound of the 95% confidence interval</th>
<th>upper bound of the 95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.200000</td>
<td>0.199846</td>
<td>0.198930</td>
<td>0.200761</td>
</tr>
<tr>
<td>0.4</td>
<td>0.399959</td>
<td>0.400041</td>
<td>0.398888</td>
<td>0.401193</td>
</tr>
<tr>
<td>0.8</td>
<td>0.766410</td>
<td>0.765848</td>
<td>0.764788</td>
<td>0.766907</td>
</tr>
<tr>
<td>0.9</td>
<td>0.781485</td>
<td>0.782117</td>
<td>0.781337</td>
<td>0.782896</td>
</tr>
<tr>
<td>1.2</td>
<td>0.749394</td>
<td>0.749384</td>
<td>0.749100</td>
<td>0.749668</td>
</tr>
</tbody>
</table>
Average System Delay, 10 nodes, B=2

![Diagram showing average system delay vs aggregated packet arrival rate for B=2, 10 nodes.]
Conclusion

• We have proposed an analytical approach for exact performance evaluation of the classic tree/stack splitting algorithm in wireless access networks with multiple packet reception and queueing