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招生學年度	100	招生類別	轉學招生考試
系所班別	資訊工程學系三年級		
科目	離散數學		
注意事項	禁止使用掌上型計算機		

Discrete Mathematics

1. (20%) Decide whether each of the following statements is true or false.

- (a) $n \lg n \in O(n)$.
- (b) $n^4 \in O(2^n)$.
- (c) $O(\lg n) = O(\lg(n^2))$.
- (d) $O(n^2) \subseteq O((\lg n)^2)$.

2. (15%) Show that $\forall n \geq 0, 6 \mid (n^3 - n)$.

3. (15%) Determine whether each of the given functions f and g are inverses of one another.

- (a) $f(x) = \sqrt[3]{x} + 1$ and $g(x) = (x - 1)^3$.
- (b) $f(x) = 4 - 2x$ and $g(x) = 2 - \frac{1}{2}x$.
- (c) $f(x) = \frac{x}{3} + 1$ and $g(x) = 3x - 3$.

4. (20%) Each of 4 black beads and 4 white beads is to be threaded on a loop to form a necklace. If the clasp will be undetectable, then how many possibilities are there for this necklace? How many possibilities are there if each of these clasps is detectable?

5. (15%) Find an NFA that accepts the language

$$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}.$$

6. (15%) We need to store chemicals in some cabinets, while being careful not to put in the same cabinet two chemicals that might cause a dangerous chemical reaction. If acids are mixed with bleach or sulfides, then a toxic gas can be produced. If acids come in contact with hydrogen peroxide, then an explosion can result. Mixing bleach with ammonia can also produce a toxic gas. What is the smallest number of cabinets needed to store these chemicals, based on dangers described?