招 生 學 年 度	102	招 生 類 別	碩士班
系 所 班 別	資訊工程學系碩士班(甲組、乙組)、資訊工程學系 網路與多媒體科技碩士班		
科 目	離散數學		
注 意 事 項	本考科禁止使用掌上型計算機		

1. (15%) The following problems refer to strings in $\{A, B, \ldots, Z\}$.

   (a) How many different four-letter strings are there?

   (b) How many four-letter strings are there that begin with A?

   (c) How many four-letter strings are there that contain exactly two Z's?

2. (15%) In a class of 17 students, 3 are math majors. A group of four students is chosen at random.

   (a) What is the probability that the group has no math majors?

   (b) What is the probability that the group has at least one math major?

   (c) What is the probability that the group has exactly two math majors?

3. (20%) Consider the following algorithm.

   $$x \leftarrow 1$$
   $$\text{for } i \in \{1, 2, \ldots, n\} \text{ do}$$
   $$\qquad \text{for } j \in \{1, 2, \ldots, n\} \text{ do}$$
   $$\qquad\qquad x \leftarrow x + 5$$
   $$\qquad \text{for } k \in \{1, 2, 3, 4, 5\} \text{ do}$$
   $$\qquad\qquad x \leftarrow x + k + 1$$

   (a) Count the number of $+$ operations done by this algorithm.

   (b) What is the value of x after the algorithm finishes?

4. (20%) Determine the following recurrence relations:

   (a) The complete graph K_n on n vertices is the undirected graph that has exactly one edge between every pair of vertices. Find a recurrence relation $E(n)$ for the number of edges in K_n.

   (b) Let $S(n) = 1^2 + 2^2 + \cdots + n^2$ be the sum of the first n perfect squares. Find a recurrence relation for $S(n)$.

5. (15%) Let $H(n)$ be defined as follows.

   $$H(n) = \begin{cases} 0 & \text{if } n \leq 0. \\ 1 & \text{if } n = 1 \text{ or } n = 2. \\ H(n-1) + H(n-2) - H(n-3) & \text{if } n > 2. \end{cases}$$

   Prove that $H(2n) = H(2n-1) = n$ for all $n \geq 1$.

6. (15%) Let X be a finite set, and let $P(X)$ be the power set of X. Let G be the graph whose vertices represent the elements of $P(X)$, where A and B are joined by an edge if $A \cap B = \emptyset$. Similarly, Let H be the graph with a vertex for each element of $P(X)$, but where A and B share an edge if $A \cup B = X$. Prove that G is isomorphic to H.