1. (15%) The running time of an algorithm A is described by the recurrence $T(n)=7T(n/2)+n^2$. A competing algorithm A' has a running time of $T'(n)=aT'(n/4)+n^2$. What is the largest integer value for a such that A' is asymptotically faster than A?
2. (15%) Show that worst-case running time of Heapify on a heap of size n is $\Omega(\lg n)$.
3. (15%) What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers ?

   a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

   Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers ?
4. (15%) Find a feasible solution or determine that no feasible solution exists for the following system of difference constraints:

   $x1-x2\leq4$, $x1-x5\leq5$, $x2-x4\leq-6$, $x3-x2\leq1$, $x4-x1\leq3$, $x4-x5\leq10$, $x5-x3\leq-4$, $x5-x4\leq-8$
5. (20%) Let X[1...n] and Y[1...n] be two arrays, each containing n numbers already in sorted order. Give an O(lgn)-time algorithm to find the median of all 2n elements in arrays X and Y.
6. (20%) Briefly explain these two algorithm design approaches: divide and conquer, dynamic programming. For each algorithm approach, give one example including a problem with an algorithm for the problem. What is the difference between them?