Ph.D. Qualification Examination

Computation Theory (Oct. 2012)

(1) (20%) Give DFA's accepting the following languages over the alphabet $\{0, 1\}$:
  (a) The set of all strings such that each block of five consecutive symbols contains at least two 0's.
  (b) The set of strings such that the number of 0's is divisible by three, and the number of 1's is divisible by two.

(2) (20%) Prove that the following are not regular languages.
  (a) $\{0^n 1 0^n \mid n \geq 1\}$.
  (b) $\{0^n \mid n \text{ is a power of } 2\}$.

(3) (20%) Determine whether the recursive languages are closed under the following operations.
  (a) Union.
  (b) Intersection.
  (c) Concatenation.
  (d) Kleene star.

(4) (20%) A two-dimensional Turing machine has the usual finite-state control but a tape that is a two-dimensional grid of cells, infinite in all directions. The input is placed on one row of the grid, with the head at the left end of the input and the control in the start state, as usual. Acceptance is by entering a final state. Prove that the languages accepted by two-dimensional Turing machines are the same as those accepted by ordinary TM's.

(5) (20%) Is each of the following problems decidable?
  (a) Is the intersection of two CFL's empty?
  (b) Are two CFL's the same?
  (c) Is $L(G)$ finite, for a given CFG G?
  (d) Does $L(G)$ contain at least 100 strings, for a given CFG G?