

Ph.D. Qualification Examination

Computation Theory (Dec. 2003)

- (1) (30%) Let  $L$  be the language defined by the grammar  $G$  of  $S \rightarrow aSbS \mid bSaS \mid \epsilon$ .
  - (a) Show that  $G$  is ambiguous.
  - (b) Find an unambiguous context-free grammar for  $L$ .
  - (c) Is it decidable whether a context-free language is ambiguous? Explain your answer.
- (2) (20%) Design a Turing machine for the language  $\{a^n b^n c^n \mid n \geq 1\}$ .
- (3) (20%) Give algorithms to decide the following:
  - (a) Is  $L(G)$  finite, for a given CFG  $G$ ?
  - (b) Given a CFG  $G$  and one of its variables  $A$ , is there any sentential form in which  $A$  is the first symbol.
- (4) (20%) Give an algorithm to tell, for two regular languages  $L_1$  and  $L_2$  over the same alphabet  $\Sigma$ , whether there is any string in  $\Sigma^*$  that is in neither  $L_1$  nor  $L_2$ .
- (5) (10%) Define the following terms.
  - (a) recursive languages
  - (b) recursively enumerable languages
  - (c) undecidable problems