

Discrete Mathematics

1. (20%) Let  $n \geq 1$ . Compute the number of ways in which  $n$  coins can be arranged in horizontal rows with all the coins in each row touching and every coin above the bottom row touching two coins in the row below it.

2. (20%) Consider the following five relations on the set  $A = \{1, 2, 3\}$ :

(a)  $\emptyset$  (empty relation)

(b)  $R = \{(1, 1), (1, 2), (1, 3), (3, 3)\}$

(c)  $S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$

(d)  $T = \{(1, 1), (1, 2), (2, 2), (2, 3)\}$

(e)  $U = A \times A$  (universal relation)

Determine whether or not each of the above relations on  $A$  is: (a) reflexive; (b) symmetric; (c) transitive; (d) antisymmetric.

3. (20%) Let  $C(n, k) = \frac{n!}{(n-k)!k!}$ . Prove that

$$C(n, 0) + C(n, 1)2 + C(n, 2)2^2 + \cdots + C(n, n)2^n = 3^n$$

4. (20%) What are the languages and the corresponding automata of Type-0, Type-1, Type-2, and Type-3? Define each language using the grammar terminologies.

5. (10%) A binary tree  $T$  has nine nodes. Draw the diagram of  $T$  if the preorder and inorder traversals of  $T$  yield the following sequences of nodes:

Preorder: G B F A C H D E K

Inorder: F B C A G H E D K

6. (10%) A barber in Cambridge, England, advertised that he "... shaves those, and only those, who do not shave themselves." Did the barber shave himself?