

1. In question a-c, please decide whether or not the limit exists. Evaluate the limits that do exist.

a. (5%) $\lim_{x \rightarrow 1} \frac{x}{x+1}$

b. (6%) $\lim_{x \rightarrow 0} \frac{\sin(4x)}{3x}$

c. (9%) $\lim_{x \rightarrow 0} \frac{x^2}{\sec(x) - 1}$

2. (10%) Differentiate $f(x) = \frac{6}{x^3} - \frac{6}{x}$

3. (15%) Calculate the derivative of $f(x) = 3x(x^2 + 3)^4$

4. (15%) Find $\int 2x^3 \sec^2(x^4 + 1) dx$

5. (20%) An architect wants to design a window in the shape of a rectangle capped by a semicircle. If the perimeter of the window is constrained to be 24 feet, what dimensions should the architect choose for the window in order to admit the greatest amount of light?

6. (20%) The region bounded by $f(x) = (6x - x^2)$ and x -axis between $x=1$ and $x=5$ is rotated about the y -axis. Find the volume of the solid that is generated.