Discrete Mathematics

1. (25%) Let R be a binary relation from A to B. The converse of R, denoted R^{-1}, is a binary relation from B to A such that $R^{-1} = \{(b, a) \mid (a, b) \in R\}$.

   (a) Let R_1 and R_2 be binary relations from A to B. Is it true that $(R_1 \cup R_2)^{-1} = R_1^{-1} \cup R_2^{-1}$.

   (b) Let R be a binary relation on A. If R is reflexive, is R^{-1} necessarily reflexive? If R is symmetric, is R^{-1} necessarily symmetric? If R is transitive, is R^{-1} necessarily transitive?

2. (15%) An ordered n-tuple $(d_1, d_2, \ldots, d_n)$ of nonnegative integers is said to be *graphical* if there exists a simple graph with no self-loops that has n vertices with the degrees of the vertices being $d_1, d_2, \ldots, d_n$.

   (a) Show that $(4, 3, 2, 2, 1)$ is graphical.

   (b) Show that $(3, 3, 3, 1)$ is not graphical.

   (c) Is $(5, 5, 3, 3, 2, 2, 2)$ graphical?

3. (20%) Seven kinds of military equipment are to be flown to a destination by five cargo planes. There are four units of each kind, and five planes can carry these kinds of equipment. Can the equipment be loaded in such a way that no two units of the the same kind are on the plane if the capacities of the five planes are as follows:

   (a) 8, 8, 5, 4, 4.

   (b) 7, 7, 6, 4, 4.

4. (20%) Draw a deterministic finite state machine that recognizes the language $L = \{(01)^i 1^{2j} \mid i \geq 1, j \geq 1\}$.

5. (20%) Given a row of n 0's and 1's, we wish to rearrange them so that the 0's will be grouped at the left and the 1's will be grouped at the right. The basic operation is to compare two adjacent digits and exchange their positions, if so desired. Design an algorithm and determine its time complexity.