

全部試題包含選擇題六題、填充題三題（六格）、計算題四題。

作答計算題時請列出計算及推導過程。

選擇題（每題五分，共三十分）

1. Evaluate  $\lim_{x \rightarrow 0} x \cot x =$  (A) 0 (B) 1 (C) 2 (D)  $\infty$ .

2. Evaluate  $\int_0^1 \int_1^{e^x} \int_0^{e^y} xy \, dz \, dy \, dx =$  (A) 0 (B)  $\frac{1}{63}$  (C)  $\frac{2}{189}$  (D)  $\frac{4}{189}$ .

3. Evaluate  $\lim_{x \rightarrow 0^+} x^x =$  (A) 0 (B)  $\frac{1}{2}$  (C) 1 (D)  $\infty$ .

4. Evaluate  $\sum_{k=1}^{\infty} \frac{2k+1}{2k^3(k+1)^3} =$  (A) 0 (B)  $\frac{1}{2}$  (C) 1 (D)  $\infty$ .

5. Which of the following series converges?

(A)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+19n}}$  (B)  $\sum_{n=1}^{\infty} \frac{n}{5n^2-4}$  (C)  $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$  (D) none of above.

6. Find the centroid of the region under the curve  $y = \sin x$ ,  $0 \leq x \leq \pi$ .

(A)  $\left(\frac{\pi}{2}, \frac{\pi}{8}\right)$  (B)  $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$  (C)  $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$  (D) none of above.

填充題（每格五分，共三十分）

7. If the functional relationship between  $x$  and  $y$  is given implicitly as  $2xy - y^3 + 1 = x + 2y$ . Use implicit differentiation to find  $dy/dx =$  \_\_\_\_\_.

8. Evaluate  $\int_0^{\frac{\pi}{2}} \sin^3 x \, dx =$  \_\_\_\_\_.

9. Let  $f(x) = \frac{\ln x}{1 + (\ln x)^2}$  for  $x$  in  $(0, \infty)$ . Find

(a)  $\lim_{x \rightarrow 0^+} f(x) =$  \_\_\_\_\_;

(b)  $\lim_{x \rightarrow \infty} f(x) =$  \_\_\_\_\_;

(c) the maximum value of  $f(x) =$  \_\_\_\_\_;

(d) if  $F(x) = \int_1^x f(t) \, dt$ , then  $F'(\sqrt{e}) =$  \_\_\_\_\_.

計算題 (每題十分，共四十分)

10. Determine the concavity and find the points of inflection (if any) of the graph of  $f(x) = x + \cos x$   $x \in [0, 2\pi]$ .

11. Find the integration  $\int \frac{\cos x}{\sin x + \sin^2 x} dx$ .

12. Test for the convergence or divergence of  $\sum_{n=1}^{\infty} \frac{\cos(n!)}{n^2}$ .

13. Use triple integration to find the volume of the solid  $T$  bounded above by the parabolic cylinder  $z = 4 - y^2$  and bounded below by the elliptic paraboloid  $z = x^2 + 3y^2$ .