

本考科禁用計算機

(請於此線以下開始出題)

1. (15%) Find the limit (if it exists). If it does not exist, explain why.

(a) (5%) $\lim_{x \rightarrow 0} \frac{|x|}{x}$ (b) (5%) $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$ (c) (5%) $\lim_{x \rightarrow 0} \frac{x^2 + x - 6}{x^2 - 9}$

2. (20%) Evaluate the following integrals.

(a) (10%) $\int_0^1 \frac{e^x}{1+e^x} dx$ (b) (10%) $\int_0^1 \frac{1}{1+e^x} dx$

3. (10%) Evaluate $\int_0^1 \int_y^1 e^{-x^2} dx dy$.

4. (15%) Let R be the region bounded by the lines

$$x - 2y = 0, x - 2y = -4, x + y = 4, \text{ and } x + y = 1.$$

Evaluate the double integral $\iint_R 3xy dA$.

5. (10%) Find the tangent plane to the graph given by $z^2 - 2x^2 - 2y^2 = 12$ at the point $(1, -1, 4)$.

6. (20%) Find the extreme values of $f(x, y) = x^2 + 2y^2 - 2x + 3$ subject to the constraint $x^2 + y^2 \leq 10$.

7. (10%) Let $f(x, y) = \int_x^y \sqrt{1+t^3} dt$. Find $f_x(x, y)$ and $f_y(x, y)$.