Discrete Mathematics

1. Suppose that in a survey of 100 students, it was found that 50 take chemistry, 55 take mathematics, 45 take physics, 18 take chemistry and physics, 20 take physics and mathematics, 24 take mathematics and chemistry, and 5 take all three.
   (a) (5%) How many students take at least one of the three areas?
   (b) (5%) How many students take none of the three areas?
   (c) (5%) How many students take mathematics only?
   (d) (5%) How many students take physics or chemistry but not mathematics?
   (e) (5%) How many students do not take either mathematics or chemistry?

2. Five boys and five girls attend a concert together.
   (a) (5%) How many ways can they sit if no boys sit together?
   (b) (5%) How many ways can they sit if no boys sit together and no girls sit together?
   (c) (5%) How many ways can they sit if all the boys sit together?
   (d) (5%) How many ways can they sit if a boy sits at each end?
   (e) (5%) How many ways can they sit if one boy and one girl refuse to sit next to each other?

3. (10%) Show that $n^3 - n$ is divisible by 3 for every positive integer n.

4. (10%) Show that if n^2 is an odd integer, then n is an odd integer.

5. (10%) Show that $\sqrt{3}$ is not a rational number.

6. True-False questions:
   (a) (4%) A component of a graph is always connected.
   (b) (4%) Every directed graph contains a vertex that is a source.
   (c) (4%) Every tree has at least two vertices of degree 1.
   (d) (4%) The spanning tree of a graph is unique.
   (e) (4%) At least half of the vertices of a tree must be leaves.