本考科禁用計算機

一 單一選擇題

1.(4%) If we sort n numbers by insertion sort, which of the following is wrong?

(a) Insertion sort has $O(n^2)$ average case performance.

(b) Insertion sort has $O(n^2)$ worst case performance.

(c) Insertion sort has $O(n)$ best case performance.

(d) None of the above

2.(4%) $\theta(n)$ and $O(n)$ actually are set of functions. But we often say $\theta(n)+O(n)$. Which one is the best to present the result of the addition?

(a) $\Omega(n)$

(b) $\theta(n)$

(c) $O(n)$

(d) None of the above

3.(4%) $\Omega(n)$ and $O(n)$ actually are set of functions. But we often say $\Omega(n)+O(n)$. Which one is the best to present the result of the addition?

(a) $\Omega(n)$

(b) $\theta(n)$

(c) $O(n)$

(d) None of the above

4.(4%) Consider the max heap in array representation:

91 63 40 38 55 28 13 18 3 45 6 22

Which will be the backmost value in the array after inserting 55?

(a) 6

(b) 18

(c) 28

(d) None of the above

二. 複選題

5.(5%)Which of the following statements are correct?

(a) If we restrict that the edges can not have equal weights, the minimum spanning tree is unique.

(b) The shortest path between two vertices is not unique.

(c) If we restrict that the edges can not have equal weights, the shortest path between two vertices is unique.

(d) Shortest path problem is equivalent to longest path

本考科禁用計算機

problem if the given digraph can have negative weights.

(e) Problem B is known to be an NP hard problem. Problem A can be proven to be NP hard by reducing A to B in an efficient way.

6.(5%)Let B be a binary search tree with n nodes. Which of the following statements are wrong?

(a) The worst case of search time is $O(\log_2 n)$.

(b) There are exactly n distinct keys in the tree.

(c) The time complexity of key insertion is $O(\log_2 n)$.

(d) The height of B is $\lfloor \log_2 n \rfloor + 1$.

(e) The height of B could be n.

7.(5%)If a node in the given B⁺-tree can store k keys, which of the following statements are wrong?

(a) The internal node has at least $\lceil (k+1)/2 \rceil$ children.

(b) B⁺-tree is a binary search tree.

(c) B⁺-tree is a balance tree.

(d) Each key appears exactly once in the tree.

(e) If there are n search keys, the worst case of search time is $O(\log_k n)$

8.(5%)Given the expression, (A+B)*C+D/(E+F*G)-H, which of the followings are true?

(a) In postfix transformation, the top of the stack is "+" after G is read.

(b) In postfix transformation, the bottom of the stack is "+" after D is read.

(c) In postfix transformation, the top of the stack is "+" after F is read.

(d) In postfix transformation, the bottom of the stack is "+" after G is read.

(e) In postfix transformation, the top of the stack is "+" after E is read.

三 問答題

9. (14%) If we can transform a directed graph without negative cycle into a directed graph that has only nonnegative weights and that has the same shortest paths structure, all pair shortest path problem can be solved in time proportional to VElogV. Please describe a reweighting method to get such a directed graph. You should justify your answer.

本考科禁用計算機

10. (10%) What are the minimum and maximum numbers of elements in a heap of height h?
11. (20%) Which of the following sorting algorithms are stable: insertion sort, merge sort, heapsort, and quicksort? Give a simple scheme that makes any sorting algorithm stable. How much additional time and space does your scheme entail?
12. (10%) What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?
    a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

    Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

13. (10%) Please construct a minimum spanning tree for the following graph using Kruskal's algorithm.