

本考科禁用掌上型計算機

1. [10%] Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.
2. [10%] Find the domain and range of these functions.
 - (a) [5%] The function that assigns to each positive integer the number of the digits 0,1,2,3,4,5,6,7,8,9 that do not appear as decimal digits of the integer.
 - (b) [5%] The function that assigns to each pair of positive integers the maximum of these two integers.
3. [15%] Show that $1^2 + 2^2 + \dots + n^2 = n(n+1)(2n+1)/6$ whenever n is a positive integer. Use mathematical induction to prove it.
4. [10%] Devise a recursive algorithm for computing the greatest common divisor of two nonnegative integers a and b with $a < b$ if $\gcd(a, b) = \gcd(a, b-a)$.
5. [15%] Please give and explain your answers to the following questions.
 - (a)[8%] How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?
 - (b)[7%] How many cards must be selected to guarantee that at least three hearts are selected?
6. [15%] Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. How many such bit strings are there of length five?
7. [10%] A bit string of length four is generated at random so that each of the 16 bit strings of length four is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 0?
8. [15%] (a)[5%] Find the binary tree of $(x-y)*z-w/s$.
(b)[10%] What are the corresponding prefix and postfix forms?

Note: Do not just give the answer, explain it.