

## 本考科禁用掌上型計算機

1. Find each integral.

(a) (5%)  $\int \frac{4}{x^2+9} dx$    (b) (5%)  $\int \frac{4x}{x^2+9} dx$    (c) (5%)  $\int \frac{1}{1+e^x} dx$ .

2. (10%) Find  $\lim_{x \rightarrow 0^+} (\sin x)^x$ .

3. (10%) Find the tangent line to the graph given by  $x^2(x^2 + y^2) = y^2$  at the point  $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ .

4. (10%) If  $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$ , find  $(f^{-1})'(0)$ .

5. (15%) Let  $R$  be the region bounded by the square with vertices  $(0, 1)$ ,  $(1, 2)$ ,  $(2, 1)$ , and  $(1, 0)$ . Evaluate the integral

$$\int_R \int (x+y)^2 \sin^2(x-y) dA.$$

6. (15%) Evaluate  $\int_1^\infty (1-x)e^{-x} dx$ .

7. Let

$$f(x, y) = \begin{cases} \frac{5x^2y}{x^3+y^3} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

Prove that

(a) (5%)  $f_x(0, 0)$  exists, and

(b) (5%)  $f$  is not differentiable at  $(0, 0)$ .

8. (15%) Find the extreme values of

$$f(x, y) = x^2 + 2y^2 - 2x + 3$$

subject to the constraint  $x^2 + y^2 \leq 10$ .