招 生 學 年 度	九十七	招 生 類 別	碩士班
系 所 班 別	資訊工程學系碩士班		
科　　　目	資料結構		
注 意 事 項	禁用計算機		

1. (a) Order the following functions by their growth rates. (5%)

   n　　n^2　　$n\log n$　　$2^{\log n}$　　2^n　　$2^{n\log n}$　　n^3　　2^{n-3}　　4^{n-3}

   (b) Let $T(n) = \begin{cases} 3 & \text{if } n = 1 \\ 2T(n-1) & \text{otherwise} \end{cases}$. Determine $O(T(n))$, and prove your answer. (5%)

2. (a) Draw a binary tree containing keys P, L, A, C, E, such that the postorder traversal visits nodes in this order: E, A, L, C, P and the inorder traversal visits nodes in order: E, L, A, P, C. (10%)

   (b) Given a sequence S containing the elements 4, 15, 6, 3, 21, and 2, insert these elements in the given order into an empty max-heap data structure. Please show the final result. (10%)

3. Write a breadth-first search algorithm and indicate the time complexity. (10%)

4. We are given n points in the unit circle, $p_i = (x_i, y_i)$, such that $0 < \sqrt{x_i^2 + y_i^2} < 1$, for $i = 1, 2, \ldots, n$. Suppose that the points are uniformly distributed; that is, the probability of finding a point in any region of the circle is proportional to the area of that region. Design and prove a $\Theta(n)$ expected-time algorithm to sort the n points by their distances $d_i = \sqrt{x_i^2 + y_i^2}$ from the origin (Hint: Design the bucket sizes in Bucket-Sort to reflect the uniform distribution of the points in the unit circle.). (10%)

5. Answer the following questions about Huffman trees.

   (a) Construct a Huffman tree for the characters: **a, b, c, d, e** with probabilities 1/16, 1/16, 1/8, 1/4, 1/2, respectively. (4%)

   (b) Let the left branch and right branch be 1 and 0, respectively. According to the Huffman tree in (a), decode the following binary sequences: (note: if the sequence cannot be decoded please indicate that "un-decoded") (6%)

   　　(b-1) 110100011101101011100,　　(b-2) 0000111100001111000010,

   　　(b-3) 00001111000011111100010.

   (c) If the weights of **a, b, c, d** and **e** are 2, 2, 4, 8, and 16, calculate the Weighted External Path Length (WEPL) for this tree. (4%)

   (d) Write a pseudo code which can find a binary tree with minimum WEPL. (8%)

6. Answer the following questions about Hash function.

   (a) What is rehashing (your answer should include how it operates and what its aim is. (4%)

   (b) A Hash function is $H_i(x) = (H_{i-1}(x) \times x) \bmod 11$, where $H_0(x) = 1$. Given input values 2, 4, 8, 9, 13, 15, 16, 19, show the result in these eleven buckets (bucket #0 – bucket #10) by rehashing operation. (4%)

   (c) Let N_i be the number of using the hash function $H_i(x)$. (6%)

   　　(c-1) Calculate $\sum_{i=1}^{2} N_i$, (c-2) Calculate $\sum_{i=1}^{4} N_i$, (c-3) Calculate $\sum_{i=1}^{6} N_i$.

   (d) When using linear probing to process the input values in (b), is the number of buckets enough or not? You should show the process of linear probing to explain your answer. (4%)

7. Suppose arrays X and Y are both sorted and, respectively, contain n elements. Give an $O(\log n)$ algorithm to find the median of $A \cup B$. (10%)