招生學年度	九十七	招生類別	碩士班
系所班別	資訊工程學系碩士班		
科目	離散數學		
注意事項	禁用計算機		

Discrete Mathematics

1. A positive integer is **perfect** if it equals the sum of its positive divisors other than itself.
   (a) (5%) Show that 28 is perfect.
   (b) (10%) Show that $2^{p-1}(2^p - 1)$ is a perfect number when $2^p - 1$ is prime.

2. (15%) Let f_k be the k-th Fibonacci number, *i.e.*, $f_k = f_{k-1} + f_{k-2}$ for $k \geq 2$ with initial conditions $f_0 = 0$ and $f_1 = 1$. Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

Show that

$$A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$$

whenever n is a positive integer.

3. (15%) Show that if f is a function from S to T where S and T are finite sets and $m = \lceil \frac{|S|}{|T|} \rceil$, then there are at least m elements of S that are mapped to the same value of T. That is, show that there are elements $s_1, s_2, \ldots, s_m$ of S such that $f(s_1) = f(s_2) = \ldots = f(s_m)$.

4. Let a_n denote the number of bit strings of length n that do not have two consecutive 0's. Let A denote the set of bit strings without two consecutive 0's.
   (a) (5%) Find a recurrence relation for a_n.
   (b) (5%) Give initial conditions for a_n.
   (c) (10%) Solve the recurrence relation for a_n.
   (d) (10%) Construct a finite-state automaton M that accepts A.

5. The **intersection graph** of a collection of sets $A_1, A_2, \ldots, A_n$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets that have a nonempty intersection.
   (a) (5%) Construct the intersection graph for the following sets: $A_1 = \{0, 2, 4, 6\}$, $A_2 = \{0, 1, 2, 3\}$, $A_3 = \{1, 3, 5, 7\}$, $A_4 = \{4, 5, 6, 7\}$, and $A_5 = \{0, 1, 6, 7\}$.
   (b) (10%) Determine whether this graph have an euler path or not.

6. (10%) Simplify the sum-of-products expansion: $xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$.