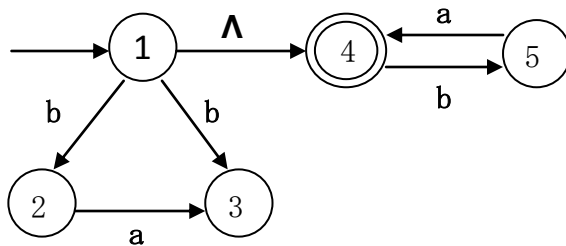


國立東華大學資訊工程系博士班資格考

計算理論, Spring 2010

1. (15%) Draw an FA that recognizes the language of all strings of 0's and 1's of length at least 1 that, if they were interpreted as binary representations of integers, would represent integers evenly divisible by 3. Your FA should accept the string 0 but no other strings with leading 0's.
2. (15%) Following figure is pictured an NFA- $\Lambda$ . Draw an NFA and a FA accepting the same language.



3. (15%) For two language  $L_1$  and  $L_2$  over  $\Sigma$ , we define the quotient of  $L_1$  and  $L_2$  to be the language  $L_1/L_2 = \{x \mid \text{for some } y \text{ in } L_2, xy \in L_1\}$ . Show that the quotient of two regular languages is regular.
4. (15%) Show that the CFG with productions:  $S \rightarrow aSaSbS \mid aSbSaS \mid bSaSaS \mid \Lambda$  generates the language  $\{x \in \{a,b\}^* \mid n_a(x) = 2n_b(x)\}$ .
5. (20%) Show that  $\{a^i b^{i+k} a^k \mid k \neq i\}$  is not a CFL.
6. (20%) Show that an infinite recursively enumerable set has an infinite recursive subset.