

招生學年度	九十八	招生類別	碩士班
系所班別	資訊工程學系		
科目	資料結構		
注意事項	本考科禁止使用掌上型計算機		

1. (15%) Solve the following recurrent relation under different conditions.

$$T(n) = \begin{cases} b & \text{if } n=1, \\ a \times T(n/c) + b \times n & \text{if } n>1. \end{cases}$$

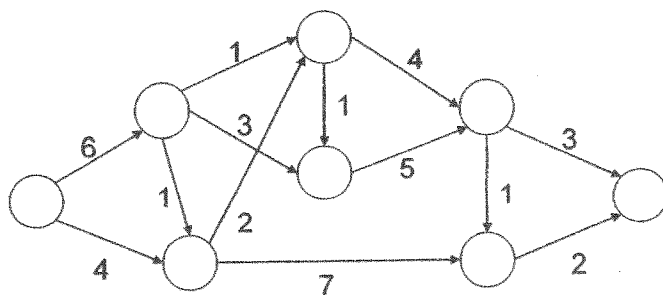
(a)  $a < c$  (5%) (b)  $a = c$  (5%) (c)  $a > c$  (5%)

2. (10%) Suppose we are given the pre-order sequence A B C D E F G H I and the in-order sequence D C E B F A H G I of the same binary tree.

(a) Draw a binary tree defined by such a pair of sequences. (5%)  
 (b) Does such a pair of sequences uniquely define a binary tree? (5%)

3. (25%) For the following AOE network  $G = (V, E)$ , please answer the questions below.

(a) Give numbers 1, 2, ..., 8 to the vertices so that for all edges  $(i, j) \in E, i < j$ . (5%)  
 (b) Obtain the early,  $e()$ , and late,  $l()$ , start times for each activity. Use the forward-backward approach. (5%)  
 (c) What is the earliest time the project can finish? (5%)  
 (d) Which activities are critical? (5%)  
 (e) Is there any single activity whose speed up would result in a reduction of the project length? (5%)



4. (30%) Given input {254, 48, 73, 199, 43, 96, 19, 51} and a hash function  $h(x) = x \text{ mod } 10$ , show the resulting:

(a) Hash table by use of chaining. (4%)  
 (b) Hash table using linear probing. (4%)  
 (c) Hash table using quadratic probing. (4%)  
 (d) Hash table by use of rehashing with  $h_i(x) = (h_{i-1}(x) + 3) \text{ mod } 10, i \geq 2$ , where  $h_1(x) = h(x)$ . (4%)  
 (e) Design a hash function, which has 8 bits at input and 4 bits at output, by a bit-XOR operation. Give a figure or equation to show your design (note: the answer is not unique). (7%)  
 (f) Redo (b) by your hash function designed in (e) (note: you should transfer the input to an 8-bit vector when use your hash function). (7%)

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5. (12%) Let  $A = \{a_0, a_1, \dots, a_{n-1}\}$  be a set of  $n$  integers such that  $a_i < a_{i+1}$  for  $0 \leq i \leq (n-2)$ . Let  $P = \{P_0, P_1, \dots, P_{k-1}\}$  be a  $k$ -partition of  $A$  such that  $|P_i| \geq 1$  and  $\max(P_i) < \min(P_{i+1})$  for  $0 \leq i \leq k-2$ , where  $\max(P_i)$  and  $\min(P_{i+1})$  denote the maximum and minimum element in  $P_i$  and  $P_{i+1}$ , respectively. Let  $B = \{b_0, b_1, \dots, b_{k-1}\}$  be a subset of  $A$  with  $b_i < b_{i+1}$  for  $0 \leq i \leq k-2$ .

(a) Prove that there at least exists an integer  $i$ ,  $0 \leq i \leq k-1$ , such that  $b_i \in P_i$ . (8%)

(b) Suppose a set  $A = \{1, 2, 5, 8, 10, 12, 13, 15, 20, 22, 25, 29, 30\}$  has 13 elements. Show an example- a partition  $P = \{P_0, P_1, \dots, P_5\}$  and  $B = \{b_0, b_1, \dots, b_5\}$ , and explain  $P$  and  $B$  satisfy the description in (a). (4%)

6. (8%) Disprove the following two statements.

(a)  $3^n = O(2^n)$ . (4%)

(b)  $n^3 2^n + 6n^2 3^n = O(n^2 2^n)$ . (4%)