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國立東華大學招生考試試題

招生學年度	九十八	招生類別	碩士班
系所班別	資訊工程學系		
科目	離散數學		
注意事項	本考科禁止使用掌上型計算機		

Discrete Mathematics

1. Let $P(x)$ be the statement "student x knows Discrete Mathematics" and let $Q(y)$ be the statement "class y contains a student who knows Discrete Mathematics." Express each of these as quantifications of $P(x)$ and $Q(y)$.

   (a) (5%) Some students know Discrete Mathematics.

   (b) (5%) Not every student knows Discrete Mathematics.

   (c) (5%) Every class has a student in it who knows Discrete Mathematics.

   (d) (5%) There is at least one class with no student who knows Discrete Mathematics.

2. (15%) Show that given a set of $n+1$ positive integers, none exceeding $2n$, there is at least one integer in this set that divides another integer in the set.

3. A **rooted spanning tree** of a directed graph is a rooted tree containing edges of the graph such that every vertex of the graph is an endpoint of one of the edges in the tree.

   (a) (10%) Show that a connected directed graph in which each vertex has the same in-degree and out-degree has a rooted spanning tree.

   (b) (10%) Propose an algorithm to construct a rooted spanning tree for connected directed graphs in which each vertex has the same in-degree and out-degree.

4. Let a_n denote the number of ways a person can climb up a ladder with n rungs. At each step he can climb one or two rungs.

   (a) (5%) Define a_n recursively.

   (b) (10%) Find an explicit formula for a_n.

5. Find the probability that a randomly generated bit string of length 10 does not contain a 0 if bits are independent and if

   (a) (5%) a 0 bit and a 1 bit are equally likely.

   (b) (5%) the probability that a bit is a 1 is 0.6.

   (c) (5%) the probability that the i-th bit is a 1 is $\frac{1}{2^i}$ for $i = 1, 2, \ldots, 10$.

6. (15%) Construct a nondeterministic finite-state automaton that recognizes the language generated by the regular grammar $G = (V, \Sigma, S, P)$, where $V = \{A, S\}$, $\Sigma = \{a, b\}$, and $P = \{S \to aA \mid b \mid \lambda, A \to aA \mid bA \mid a\}$.