

# Ph.D. Qualification Examination

## Algorithms (April 2009)

- (1) (20%) Solve the following recurrences. Assume that  $T(c) = 1$  for a constant  $c$ .
  - (a)  $T(n) = T(\frac{n}{2}) + \log n$
  - (b)  $T(n) = T(\sqrt{n}) + \log n$
- (2) (20%) Define a decision version of the sorting problem. Then, propose a nondeterministic polynomial-time algorithm for this decision sorting problem. What is the running time of your algorithm?
- (3) (20%) Let  $G = (V, E)$  be any connected undirected graph. A *cut vertex* of  $G$  is a vertex  $v$  such that the graph obtained from  $G$  by removing  $v$  is disconnected. Give an efficient algorithm to find all the cut vertices of  $G$ . What is the running time of your algorithm?
- (4) (20%) Given a connected graph  $G = (V, E)$  and a minimum spanning tree  $T$  of  $G$ , propose an efficient algorithm to find a second best spanning tree  $T'$  of  $G$  if  $T'$  exists. What is the running time of your algorithm?
- (5) (20%) Given a text  $T$  and two integers  $l, k$ , propose an algorithm to find all the patterns  $P$  such that  $|P| \geq l$  and  $P$  appears in  $T$  at least  $k$  times. What is the running time of your algorithm?