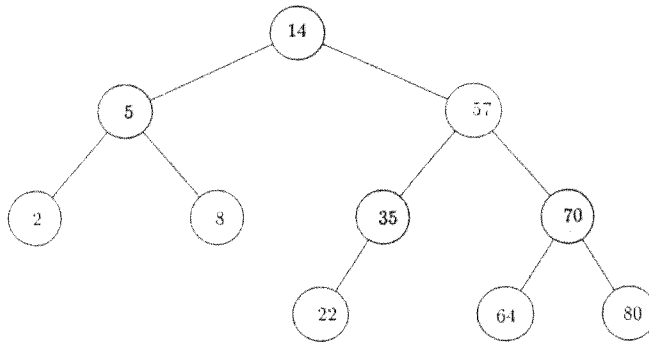


NDHU CSIE PhD Qualification Exam Algorithms, Fall 2007

- (1) (20%) Answer with **True** or **False** and briefly explain your answer. Your answer will be considered wrong if you fail to provide a satisfactory explanation.
- $f(n) = O(g(n))$ implies $g(n) = O(f(n))$
 - $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$
 - $\Omega(f(n)) \subset \Theta(f(n))$
 - $\sin(f(n)) = O(0.5)$
 - $n^{\frac{1}{1000}} = O(\log(n))$
 - $n^{1000} = \Omega(1.01^n)$
 - $n! = O(n^n)$
- (2) (10%) Solve the following recurrences.
- $T(n) = T(n-1) + \log(n)$ (express the solution as $T(n) = \Theta(f(n))$.)
 - $$\begin{cases} T(n) = T(\sqrt{n}) + 1 & n > 2 \\ T(n) = 1 & n \leq 2 \end{cases}$$
 (express the solution as $T(n) = O(f(n))$.)
- (3) (10%) Consider the following red-black tree (shaded nodes are *black* and unshaded nodes are *red*). How does the tree look like after inserting 60? Draw the resulting tree.



- (4) (20%) Huffman codes are a widely used and very effective technique for compressing data using binary symbols. Generalize Huffman's algorithm to ternary codewords (*i.e.*, codewords using the symbols 0, 1, and 2). Analyze the time complexity of your algorithm.
- (5) (15%) Let $X[1..n]$ and $Y[1..n]$ be two arrays, each containing n numbers already in sorted array. Give an algorithm to find the median of all $2n$ elements in arrays X and Y . Analyze the time complexity of your algorithm.
- (6) (10%) A certain problem has been shown that it is NP-hard. Which of the following assertions is true?
- The problem cannot be solved in polynomial time.
 - The lower bound (time complexity) of the problem is exponential.
 - The problem is intractable.
 - All of the above.
- (7) (15%) Suppose that a graph G has a minimum spanning tree already computed. How quickly can the minimum spanning tree be updated if a new vertex and incident edges are added to G .