

1. (5%) Please draw an FA recognizing the language for the regular expression $(11+10)^*$.
2. (10%) Suppose that M is an NFA- Λ accepting $L \subseteq \Sigma^*$. Describe how to modify M to obtain an NFA- Λ recognizing $rev(L) = \{x^r \mid x \in L\}$.
3. (15%) Decide whether each statement below is true or false. If it is true, prove it. If not, give a counterexample. All parts refer to languages over the alphabet $\{0,1\}$.
 - (a) \cdot If L_1 and L_2 are nonregular, then $L_1 \cup L_2$ is nonregular.
 - (b) \cdot If L_1 is regular, L_2 is nonregular and $L_1 \cap L_2$ is regular, then $L_1 \cup L_2$ is nonregular.
 - (c) \cdot If L_1, L_2, L_3, \dots are all regular, then $\bigcup_{n=1}^{\infty} L_n$ is regular.
4. (10%) Decide whether each language over $\{0,1\}$ below is regular, and prove that your answer is correct.
 - (a) \cdot The set of odd-length strings over $\{0,1\}$ with middle symbol 0.
 - (b) \cdot The set of strings x for which there is an integer $k > 1$ (possibly depending on x) so that the number of 0's in x and the number of 1's in x are both divisible by k .
5. (10%) Show that the CFG with productions $S \rightarrow a \mid Sa \mid bSS \mid Ssb \mid Sbs$ is ambiguous.
6. (10%) Please explain the following terms:
 - (a) \cdot Pushdown Automata
 - (b) \cdot Turing Machines
 - (c) \cdot Recursively Enumerable Languages
 - (d) \cdot Recursive Languages
 - (e) \cdot Context-Sensitive Grammars
7. (10%) Give the transition table for the PDA recognizing the language $\{a^n x \mid n \geq 0, x \in \{a,b\}^* \text{ and } |x| \leq n\}$.
8. (10%) Use the pumping lemma to show that the language $L = \{x \in \{a,b,c\}^* \mid n_a(x) = \max\{n_b(x), n_c(x)\}\}$ is not a CFL.
9. (10%) Draw a transition diagram for a TM accepting the language $\{a^n b^n c^n \mid n \geq 0\}$.
10. (10%) Is it true that if L_1 and L_2 are recursively enumerable subsets of Σ^* then $L_1 \cup L_2$ is recursively enumerable? If yes, prove it; otherwise, disprove it.