Ph.D. Qualification Examination

Computation Theory (Oct. 2004)

(1) (25%) Let L_1 and L_2 be two decidable languages. Prove or disprove
   (a) $L_1 \cup L_2$ is decidable.
   (b) $L_1 \cap L_2$ is decidable.
   (c) $\overline{L_1}$ is decidable.
   (d) $L_1 L_2$ is decidable.
   (e) $(L_1)^*$ is decidable.

(2) (25%) Let L_1 and L_2 be two languages over a fixed alphabet Σ with $|\Sigma| = 2$. Justify your answers for the following questions.
   (a) If L_1 is nonregular and $L_1 \subseteq L_2$, then L_2 is nonregular.
   (b) $L_1 \subseteq L_2$ and L_2 is nonregular, then L_1 is nonregular.
   (c) If L_1 is nonregular, then its complement $\overline{L_1}$ is nonregular.
   (d) L_1 is regular, then $L_1 \cup L_2$ is regular for any language L_2.
   (e) L_1 and L_2 are nonregular, then $L_1 \cap L_2$ is nonregular.

(3) (20%) Design a Turing machine for the language $\{ww^R \mid w \in \{a, b\}^*\}$.

(4) (30%) Determine whether or not the following languages are context-free.
   (a) $L = \{a^n b^j a^k b^l \mid n + j \leq k + l\}$
   (b) $L = \{w_1 c w_2 \mid w_1, w_2 \in \{a, b\}^*,\ w_1 \neq w_2\}$
   (c) $L = \{a^{nm} \mid n \text{ and } m \text{ are prime numbers}\}$