# Ph.D. Qualification Examination

Computation Theory (Apr. 2005)

(1) (15%) For any string $w = w_1 w_2 \ldots w_n$, the *reverse* of $w$, written $w^R$, is the string $w$ in reverse order, $w_n \ldots w_2 w_1$. For any language $A$, let $A^R = \{w^R \mid w \in A\}$. Show that if $A$ is regular, so is $A^R$.

(2) (10%) Prove or disprove that $B = \{a^{2^n} \mid n \geq 0\}$ is a regular language.

(3) (20%) Determine whether the following languages are context-free or not.
  (a) $C_1 = \{x\#y \mid x, y \in \{0,1\}^* \text{ and } x = y\}$
  (b) $C_2 = \{x\#y \mid x, y \in \{0,1\}^* \text{ and } x \neq y\}$

(4) (15%) Show that, if $G$ is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of $w$.

(5) (20%) Show that the collection of Turing-recognizable languages is closed under the operations of
  (a) union.
  (b) concatenation.
  (c) star.
  (d) intersection.

(6) (20%) Consider the problem of testing whether a Turing machine $M$ on an input $w$ ever attempts to move its head left at any point during its computation on $w$. Formulate this problem as a language and show that it is decidable.