Ph.D. Qualification Examination Algorithms Fall 2010

1. (20%) Answer the following questions about hash tables:

(a) (10%) Consider inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 into a hash table of length m = 11 using open addressing with the auxiliary hash function h'(k) = k mod m. Illustrate the result of inserting these keys using double hashing with $h_2(k) = 1 + (k \bmod (m-1))$.

(b) (10%) Consider an open-address hash table with uniform hashing. Give upper bounds on the expected number of probes in an unsuccessful search and on the expected number of probes in a successful search when the load factor is 3/4 and when it is 7/8.

2. (10%) Show how to implement a stack using two queues. Analyze the running time of the stack operations.

3. (20%) We are given n points in the unit circle, $p_i = (x_i, y_i)$, such that $0 < \sqrt{x_i^2 + y_i^2} < 1$, for $i = 1, 2, \ldots, n$. Suppose that the points are uniformly distributed; that is, the probability of finding a point in any region of the circle is proportional to the area of that region. Design and prove a $\Theta(n)$ expected-time algorithm to sort the n points by their distances $d_i = \sqrt{x_i^2 + y_i^2}$ from the origin (Hint: Design the bucket sizes in Bucket-Sort to reflect the uniform distribution of the points in the unit circle.).

4. (10%) Explain the following terms:

   a. Approximation algorithm
   b. NP-complete
   c. NP-hard
   d. co-NP
   e. complexity class P

5. (20%) Briefly describe Huffman's algorithm. Then, what is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers ?

a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

6. (20%) Let e be a maximum-weight edge on some cycle of a graph G=(V,E). Prove that there is a minimum spanning tree of G=(V, E-e) that is also a minimum spanning tree of G. That is, there is a minimum of G that does not include e.