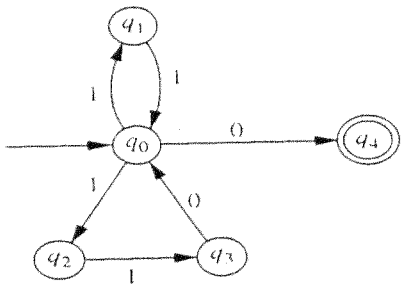
國立東華大學資訊工程系博士班資格考
計算理論, Fall 2007

1. (10%) Prove the formula $(aa^*bb^*)^* = \Lambda + a(a+b)^*b$

2. (20%) For each statement below, decide whether it is true or false. If it is true, prove it. If not, give a counterexample. All parts refer to language over the alphabet {0,1}
   (a) If L is nonregular, then L′ is nonregular.
   (b) L_1 is regular, L_2 is nonregular, and $L_1 \cap L_2$ is nonregular, then $L_1 \cup L_2$ is nonregular.

3. (20%) Describe a clear method that converts an NFA into a DFA. Use the following graph as an example to demonstrate how your method converts it into a DFA.



4. (20%) Decide in each case whether the given language is a CFL, and prove your answer.
   (a) L1={ambnambn|m≥1 and n≥1}
   (b) L2={w|w∈{a,b,c}*,na(w)< nb(w) <nc(w)}

5. (20%) Decide in each case whether the given problem is solvable, and prove your answer.
   (a) "Given two TMs T1 and T2, is L(T1)⊆L(T2)?"
   (b) "Given a CFG G with terminal alphabet ∑, is L(G)= ∑*?"

6. (10%) Show that any subset of a countable set is countable.