Arbitrary-State Attribute-Based Encryption with Dynamic Membership

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Traditional PKI has some features:

- A sender should get the public key (or the identity) of the receiver in advance.
- The more receivers the system has, the more bandwidth it consumes.
- Broadcast encryption may be able to solve the problem of performance, but the sender needs to have the receiver list.
In 1993, Fiat, and Naor proposed a broadcast encryption system.

In 2001, Boneh and Franklin proposed an identity-based encryption (IBE) scheme based on Weil pairing, and enhanced it by the technique from Fujisaki and Okamoto.

In 2005, Sahai and Waters proposed a new type of IBE, fuzzy IBE, which was the prototype of ABE.

In 2007, Baek, Susilo, and Zhou presented another fuzzy IBE with new construction.
A melancholia patient

He wants to share the experiences of struggling against melancholia on a platform with doctors or with other male patients

Attribute:

\( \text{(("Gender: MALE" AND "MENTAL DISORDER: MELANCHOLIA") OR ("CAREER: DOCTOR" AND "SPECIALITY: MELANCHOLIA"))} \)

Our scheme is the first one which supports dynamic membership and arbitrary-state attributes.

It is CCA secure under a standard model (without using random oracles).
Dynamic Membership

- **Expandability.** A new user is able to enroll in the system.

- **Renewability.** Each user’s private key, attribute set, and attribute values can be renewed and the old private keys should be useless to those ciphertexts which are encrypted after these parameters associated with the user are renewed.

- **Revocability.** A user’s private key can be revoked and the revoked private keys should be useless to those ciphertexts which are encrypted after these private keys are revoked.

- **Independence.** When a user’s leaving or attribute updating occurs, the other users are not required to interact with KGC (Key Generation Center) to renew their private keys.
### Lagrange Interpolation

Lagrange interpolating polynomial is a polynomial $p$ of degree not greater than $(n - 1)$ that passes through $n$ points $(x_i, y_1), \ldots, (x_n, y_n)$, and is given by

$$p(x) = \sum_{j=1}^{n} p_j(x), \text{ where } p_j(x) = y_j \prod_{k=i, \ldots, n, k \neq j}^{x} \frac{x - x_k}{x_j - x_k}.$$  

For $i \in \mathbb{Z}$ and $S \subseteq \mathbb{Z}$, the Lagrange coefficient $\Delta_{i,S}(x)$ is defined as

$$\Delta_{i,S}(x) = \prod_{\forall j \in S, j \neq i}^{x} \frac{x - j}{i - j}.$$
Let $G_0$, $G_1$, and $G_T$ be three cyclic groups of prime order $q$. A bilinear mapping $e : G_0 \times G_1 \to G_T$ satisfies the following properties:

- **Bilinearity:** $e(aP, bQ) = e(P, Q)^{ab}$, $\forall P \in G_0$, $Q \in G_1$ and $a, b \in \mathbb{Z}_q$.

- **Non-Degeneracy:** The mapping does not map all pairs in $G_0 \times G_1$ to the identity in $G_T$.

- **Computability:** There is an efficient algorithm to compute $e(P, Q)$, $\forall P \in G_0$, $Q \in G_1$. 
Decisional Bilinear Diffie-Hellman Problem

Let $G_0, G_1,$ and $G_T$ be three cyclic groups of prime order $q$, $P$ and $Q$ be arbitrarily-chosen generators of $G_0$ and $G_1$, respectively, and $e : G_0 \times G_1 \rightarrow G_T$ be a bilinear mapping. Given $(P, Q, aP, bP, cP, aQ, bQ, cQ, Z)$ for some $a, b, c \in \mathbb{Z}_q^*$ and $Z \in_R \{e(P, Q)^{abc}, Y \in_R G_T\}$, decide if $Z = e(P, Q)^{abc}$. 
Definition 1 (The DBDH Assumption)

We define that an adversary \( C \) with an output \( b' \in \{0, 1\} \) has advantage \( \epsilon' \) in solving the DBDH problem if

\[
\left| \Pr[C(P, Q, aP, bP, cP, aQ, bQ, cQ, e(P, Q)^{abc}) = 1] - \Pr[C(P, Q, aP, bP, cP, aQ, bQ, cQ, Z) = 1] \right| \geq \epsilon'
\]

where the probability is over the random choice of \( a, b, c \in \mathbb{Z}_q^* \) and the random choice \( Z \in \{e(P, Q)^{abc}, Y \in_R G_T\} \).
The Proposed Scheme
Overview: Enrollment, Encryption, Decryption

**Setup** → **KGC** → **Enrollment**

- **Setup** → **KGC**: \(k\), \((PK, MK)\)
- **KGC** → **Enrollment**: \((A_i, (m_{i,j})_{\forall j \in A_i}, MK)\)
- **Enrollment** → **User i**: \(D_i\) and updated \(PK\)
- **User i** → **Encryption**: \((T, M, PK)\)
- **Encryption** → **User j**: \(CT\)
- **User j** → **Decryption**: \((CT, D_j)\)
- **Decryption** → **User j'**: \(M\)
- **User j'**
The Proposed Scheme

Overview: Leaving, Updating

Leaving

User $u$

Update an attribute $D'_i$

KGC

$m'_{i,j}$

$(D'_i, PK')$

Updating

User $i$

Update an attribute

$PK'$

Leaving
The Proposed Scheme

Setup($k$)

- **Step 1:** $G_0$, $G_1$, $G_T$ with prime order $q$, $e : G_0 \times G_1 \rightarrow G_T$, generator $P$ of $G_0$, generator $Q$ of $G_1$
- **Step 2:** $\alpha, \beta \in R \mathbb{Z}_q^*$ and $H : \{0, 1\}^* \rightarrow \mathbb{Z}_q^*$.
- **Step 3:** Generate two default users:
  1. $U = \{0, 1\}$, $\{v_{i,j}\}_{\forall i \in U, j \in A} \in R \mathbb{Z}_q^*$, $\{t_i\}_{\forall i \in U} \in R \mathbb{Z}_q^*$.
  2. $
  \begin{cases} 
  \{V_j = (\prod_{\forall i \in U} v_{i,j})Q\}_{\forall j \in A} \\
  \{\overline{v_{i,j}} = t_i \prod_{\forall k \neq i, k \in U} v_{k,j}^{-1} + v_{i,j} \mod q\}_{\forall i \in U, \forall j \in A}.
  \end{cases}
  $
- **Step 4:** Public parameter $PK = (G_0, G_1, G_T, e, H, P, Q, U = e(P, Q)^{\alpha(\beta - 1)}, e(P, Q)^{\alpha\beta}, \{V_j, \{\overline{v_{i,j}}\}_{\forall i \in U}\}_{\forall j \in A, \forall})$
  The master key of KGC is $MK = \alpha Q$.

The asymmetric setting of $e$ can be implemented by BN-Curve which is with faster software implementations.
The Proposed Scheme
Enrollment($A_i$, $(m_{i,j})_{\forall j \in A_i}$, $MK$)

- **Step 1:** $r_i, t_i, \{v_{i,j}\}_{\forall j \in A}, \{r_{i,j}\}_{\forall j \in A_i} \in R \mathbb{Z}_q^*$
- **Step 2:** $U = U \cup \{i\}$
- **Step 3:**
  
  $$\begin{cases} 
  \{h_{i,j} = H(m_{i,j})\}_{\forall j \in A_i} \\
  \{V_j = v_{i,j} V_j\}_{\forall j \in A} \\
  \{\overline{v_{i,j}} = t_i \prod_{\forall k \neq i, k \in U} v_{k,j}^{-1} + v_{i,j} \mod q\}_{\forall j \in A} \\
  \{\overline{v_{k,j}} = (\overline{v_{k,j}} - v_{k,j}) v_{i,j}^{-1} + v_{k,j} \mod q\}_{\forall k \neq i, k \in U, \forall j \in A}
  \end{cases}$$

- **Step 4:** Generate user $i$’s private key $D_i =$

  $$\begin{cases} 
  D_i = \alpha Q + t_i r_i Q \\
  \{D_{i,j} = v_{i,j}^{-1} (r_i P + r_{i,j} h_{i,j} P)\}_{\forall j \in A_i} \\
  \{D_{i,j}' = t_i r_{i,j} P\}_{\forall j \in A_i} \\
  \{D_{i,j}'' = r_i P + r_{i,j} h_{i,j} P\}_{\forall j \in A_i}
  \end{cases}$$

- **Step 5:** Increase $\mathbb{V}$ and update $(\{V_j, \{\overline{v_{i,j}}\}_{\forall i \in U}\}_{\forall j \in A}, \mathbb{V})$ in $PK$. 
KGC increases $\forall$ and updates $PK$ as follows:

$$\left\{ \begin{array}{l}
\{ V_j = v_{u,j}^{-1} V_j \} \forall j \in A \\
\{ v_{k,j} = (v_{k,j} - v_{k,j}) v_{u,j} + v_{k,j} \mod q \} \forall k \neq u, k \in U, \forall j \in A
\end{array} \right.$$  

Finally, it sets $U = U \setminus \{ u \}$ and deletes $\{ \overline{v_{u,j}} \} \forall j \in A$ in $PK$. 
The Proposed Scheme

Updating \((m_{i,j}')\)

- **Step 1:** 
  \[ v_{i,j}', r_{i}', r_{i,j}' \in R \mathbb{Z}_q^* \]

- **Step 2:** 
  \[ h_{i,j}' = H(m_{i,j}') \]

- **Step 3:** Give

\[
\begin{align*}
D_i &= \alpha Q + t_{i}r_{i}' Q \\
D_{i,j} &= v_{i,j}^{-1}(r_{i}' P + r_{i,j}' h_{i,j}' P) \\
\{D_{i,k} &= v_{i,k}^{-1}(r_{i}' P + r_{i,k} h_{i,k} P)\} \forall k \in A_i \setminus \{j\} \\
D_{i,j}' &= t_{i}r_{i,j}' P \\
D_{i,j}'' &= r_{i}' P + r_{i,j}' h_{i,j}' P \\
\{D_{i,k}' &= r_{i}' P + r_{i,k} h_{i,k} P\} \forall k \in A_i \setminus \{j\}
\end{align*}
\]

\(\) to user \(i\).

- **Step 4:** Increase \(\mathbb{V}\) and update \(PK\).

\[
\begin{align*}
V_{j} &= v_{i,j} v_{i,j}' V_{j} \\
v_{i,j} &= (v_{i,j} - v_{i,j}) + v_{i,j}' \mod q \\
v_{k,j} &= (v_{k,j} - v_{k,j}) v_{i,j} v_{i,j}'^{-1} + v_{k,j} \mod q, \forall k \in \mathcal{U} \setminus \{i\}
\end{align*}
\]
Access tree structure construction.

- **For the root node** $R$:
  1. $s \in \mathbb{Z}_q^*$ and $k_R$ is the threshold value of $R$.
  2. Randomly choose a polynomial $q_R$ of degree $d_R = k_R - 1$ with $q_R(0) = s$.
  3. Assign a unique index number $x$ for each child of $R$.

- **For each internal node** $N$ **other than** $R$:
  1. $k_N$ is the threshold value of $N$.
  2. Randomly choose a polynomial $q_N$ of degree $d_N = k_N - 1$ with $q_N(0) = q_{parent(N)}(index(N))$.
  3. Assign a unique index number $x$ for each child of node $N$.

- **For each leaf node** $N_L$: Randomly choose a polynomial $q_{N_L}$ of degree 0 with $q_{N_L}(0) = q_{parent(N_L)}(index(N_L))$. 


The Proposed Scheme

Encryption

Ciphertext generation: \( K \in_R G_T \) and \( \mathcal{N}_L = \{\text{the leaves of } T\} \).

The ciphertext is

\[
CT = (T, \tilde{C} = e(P, Q)^{\alpha \beta s} K, C = sP, C'' = U^s, \\
\overline{M} = E_K(M), C_r = H(K||\overline{M})P, \\
\{C_N = q_N(0)V_{\text{att}(N)}, C'_N = q_N(0)H(\text{val}(N))Q, \\
\{v_{i, \text{att}(N)}\}_{\forall i \in \mathcal{U}}\}_{\forall N \in \mathcal{N}_L, \forall V}).
\]

\( \{\{v_{i, \text{att}(N)}\}_{\forall i \in \mathcal{U}}\}_{\forall N \in \mathcal{N}_L} \) can be excluded from the ciphertext.
The Proposed Scheme
Decryption \((CT, D_i)\)

**DecryptNode** \((CT, D_i, N)\): Let \(V_j = v_j P\).

- **If \(N\) is a leaf node:** Let \(j = att(N)\). If \(j\) is in \(A_i\) and \(m_{i,j} = val(N)\), then

\[
\begin{align*}
\text{DecryptNode}(CT, D_i, N) &= e(D_{i,j}, v_{i,j} C_N) \\
&= e(D'_{i,j}, C'_N) e(D''_{i,j}, C_N) \\
&= e(v_{i,j}^{-1}(r_i P + r_{i,j} h_{i,j} P), (t_i v_{i,j}^{-1} + v_{i,j}) q_N(0) v_j Q) \\
&= e(t_i r_{i,j} P, q_N(0) h_{i,j} Q) e(r_i P + r_{i,j} h_{i,j} P, v_j q_N(0) Q) \\
&= e(v_{i,j}^{-1}(r_i P + r_{i,j} h_{i,j} P), t_i v_{i,j} q_N(0) + v_{i,j} v_j q_N(0) Q) \\
&= e(t_i r_{i,j} P, q_N(0) h_{i,j} Q) e(r_i P + r_{i,j} h_{i,j} P, v_j q_N(0) Q) \\
&= e(r_i P, t_i q_N(0) Q) e(r_{i,j} h_{i,j} P, t_i q_N(0) Q) \\
&= e(P, Q) t_i r_i q_N(0).
\end{align*}
\]

Otherwise, \(\text{DecryptNode}(CT, D_i, N) = \bot\).
The Proposed Scheme

Decryption

- **If** \( N \) **is an internal node:**
  1. For each child \( N_c \) of \( N \), \( F_{N_c} = DecryptNode(CT, D_i, N_c) \).
  2. Let \( I_c \) be a \( k_N \)-sized set containing the indexes of the child nodes \( N_c \)'s such that \( F_{N_c} \neq \perp \) for each \( N_c \). Return

\[
F_N = \prod_{\forall \text{index} (N_c)=z \in I_c} F_{N_c}^{\Delta z, I_c(0)}
\]

\[
= \prod_{\forall \text{index} (N_c)=z \in I_c} (e(P, Q)^{t_ir_i q_{N_c}(0)})^{\Delta z, I_c(0)}
\]

\[
= \prod_{\forall \text{index} (N_c)=z \in I_c} (e(P, Q)^{t_ir_i q_{\text{parent}(N_c)(z)})^{\Delta z, I_c(0)}
\]

\[
= \prod_{\forall \text{index} (N_c)=z \in I_c} (e(P, Q)^{t_ir_i q_{N}(z)})^{\Delta z, I_c(0)}
\]

\[
= e(P, Q)^{t_ir_i q_{N}(0)}
\]

3. If no such set exists, \( N \) is not satisfied and return \( F_N = \perp \).
The Proposed Scheme

Decryption

Call \( DecryptNode(CT, D_i, R) \):

\[
A = DecryptNode(CT, D_i, R) \\
= e(P, Q)^{t_i r_i q R(0)} \\
= e(P, Q)^{t_i r_i s}
\]

Compute the session key \( K \):

\[
\frac{A \cdot \tilde{C}}{e(C, D_i) \cdot C'} = \frac{e(P, Q)^{t_i r_i s} \cdot e(P, Q)^{\alpha \beta s} K}{e(sP, (\alpha + t_i r_i)Q) \cdot e(P, Q)^{\alpha (\beta - 1)s}} \\
= \frac{e(P, Q)^{\alpha \beta s} \cdot e(P, Q)^{t_i r_i s} K}{e(P, Q)^{(\alpha s + \alpha (\beta - 1)s + t_i r_i s)}} \\
= \frac{e(P, Q)^{\alpha \beta s + t_i r_i s} K}{e(P, Q)^{\alpha \beta s + t_i r_i s}} \\
= K
\]

The decryption procedure will return \( \bot \) if \( C_r \neq H(K || \overline{M}) P \). Otherwise, it returns \( M \) by computing \( M = D_K(\overline{M}) \).
The *Enrollment* algorithm:

1. Expandability

The *Leaving* and *Updating* algorithms:

1. Revocability
2. Renewability
3. Independence
Theorem

The proposed CP-ABE-DM scheme is $CCA_{dm}$ secure under the DBDH assumption in a standard model.
Security Proof

The $CCA_{DM}$ Game

The Decisional Bilinear Diffie-Hellman Problem

$(\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T, q, P, Q, aP, bP, cP, aQ, bQ, cQ, Z)$

$PK = (\mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T, e, H, P, Q, U = e(aP, bQ - Q), e(aP, bQ))$

$MK = aQ$

$A$ outputs $b''' \in \{0, 1\}$

$C$ solves the Decisional Bilinear Diffie-Hellman problem with non-negligible advantage.

If $b''' = b''$, $C$ outputs $b' = 1$, otherwise $b' = 0$
Security Proof
The Cipher Algorithm

\[ \text{Cipher}(T, N, C_0) \]
1. Locate the node \( N \) in \( T \);
2. If \( (N \) is a leaf node) \{ 
3. \quad \text{Set } j = \text{att}(N) \text{ and } m = \text{val}(N); 
4. \quad \text{Compute } u_m = H(m); 
5. \quad \text{Compute } C_N = v_j C_0; \quad \text{// } v_j = \prod_{i \in U} v_{i,j} 
6. \quad \text{Compute } C'_N = u_m C_0; 
7. \quad \text{Store } (C_N, C'_N) \text{ in } \mathcal{L}_C; \}
8. Else \{ 
9. \quad \text{Randomly select } k_N - 1 \text{ elements } c_i \in \mathbb{Z}_q^*; 
10. \quad \text{For each child } N_C \text{ of the node } N \{ 
11. \quad \quad \text{Set } x = \text{index}(N_C); 
12. \quad \quad \text{If } (k_N - 1 > 0) \{ 
13. \quad \quad \quad \text{Compute } \overline{C}_0 = C_0 + \sum_{i=1}^{k_N-1} c_i x^i Q; \}
14. \quad \quad \text{Else } \{ 
15. \quad \quad \quad \text{Set } \overline{C}_0 = C_0; \}
16. \quad \quad \text{Call } \text{Cipher}(T, N_C, \overline{C}_0); \}
\}

Initially, \( C \) calls \( \text{Cipher}(T^*, R, cQ) \).
Comparisons

- **Dynamic Membership**: It allows an ABE system to manage member enrollment, attribute updating, and member revocation efficiently.

- **Sender Updating**: A sender must grab the newest public information before she/he encrypts a message.

- **Receiver Updating**: A receiver must interact with the system to refresh her/his private key or retrieve the newest public information before she/he decrypts a ciphertext.

- **No Private Key Refreshment**: Members do not have to interact with the system to refresh their private keys when the membership or any of the members’ attributes has been changed.

- **Arbitrary-State Attribute**: The domain of each attribute is a variable-length string, not a binary bit only.
## Feature Comparisons

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Dynamic Membership</th>
<th>ASA</th>
<th>Special Feature</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Updating</td>
<td>Leaving</td>
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<tr>
<td>Ours</td>
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<td>[4]</td>
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# Feature Comparisons

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ASA: Arbitrary-State Attribute
### Notations for Performance Comparisons

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>the number of the members in an ABE system</td>
</tr>
<tr>
<td>( m )</td>
<td>the number of the attributes provided in an ABE system</td>
</tr>
<tr>
<td>( m_c )</td>
<td>the number of attributes associated to a ciphertext</td>
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<tr>
<td>( m_d )</td>
<td>the maximum of (</td>
</tr>
<tr>
<td>( m_u )</td>
<td>the number of a user’s attributes</td>
</tr>
<tr>
<td>( m_a )</td>
<td>the number of authorities</td>
</tr>
<tr>
<td>( m_{or} )</td>
<td>the number of OR gates of the access rule associated to a ciphertext</td>
</tr>
<tr>
<td>( m_{and} )</td>
<td>the number of the AND conjunction attributes of the access rule associated to a ciphertext</td>
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<tr>
<td>( m_{not} )</td>
<td>the number of the NOT conjunction attributes of the access rule associated to a ciphertext</td>
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<tr>
<td>DR, IR</td>
<td>Direct Mode and Indirect Mode</td>
</tr>
<tr>
<td>SA, OA</td>
<td>Subject Attribute and Objected Attribute</td>
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### Performance Comparisons: Computation Cost

<table>
<thead>
<tr>
<th></th>
<th>Encryption Cost of the Sender</th>
<th>Decryption Cost of the Receiver</th>
<th>The Necessary Computation Cost of the Center</th>
<th>Enrollment</th>
<th>Updating</th>
<th>Leaving</th>
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</thead>
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<tr>
<td>Ours</td>
<td>$\mathcal{O}(m_c)$</td>
<td>$\mathcal{O}(m_c)$</td>
<td></td>
<td>$\mathcal{O}(m + m_u)$</td>
<td>$\mathcal{O}(m + m_u)$</td>
<td>$\mathcal{O}(m)$</td>
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<tr>
<td>[4]</td>
<td>$\mathcal{O}(m \times m_c + m_d)$</td>
<td>$\mathcal{O}(m_c)$</td>
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## Performance Comparisons: Storage and Communication Cost

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</table>
An attribute-based encryption scheme with dynamic membership has been proposed.

This is the first ABE scheme which can support arbitrary-state attributes and attribute (and value) updating with Sender Updating only.

It has been formally proved to be CCA secure under a standard model.
Thanks for Listening